Computer Graphics

5 - Vertex Processing 1

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Spring 2025

Outline

- Rasterization Pipeline & Vertex Processing
- Modeling Transformation
- Viewing Transformation

Rasterization Pipeline & Vertex Processing

Recall: Rasterization Pipeline

Vertex Processing
: transforms vertices to screen space

Primitive Processing
: assembles polygons

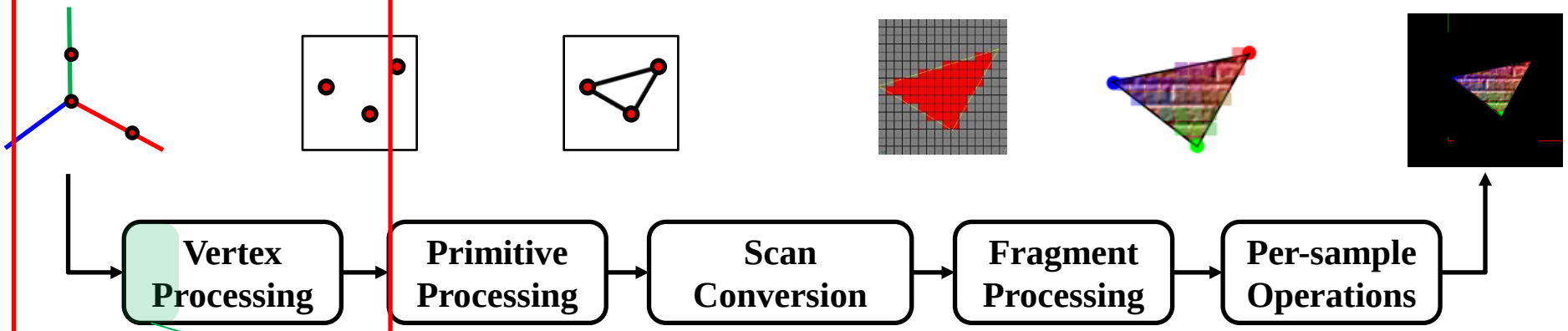
Scan Conversion
: converts each polygon into a set of fragments

Fragment Processing
: determines color of each fragment with light & texture

Per-sample Operations
: depth test, alpha blending, …

* The image is from https://graphicscompendium.com/intro/01-graphics-pipeline

Recall: Rasterization Pipeline



: transforms vertices to screen space

→ a sequence of vertex transformations

What we've been done so far

→ What we'll see today & next lecture

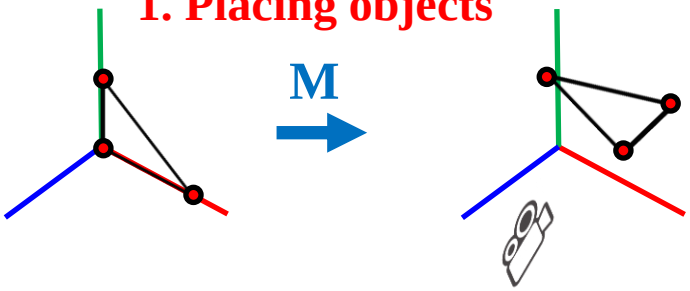
Vertex Processing

Vertex positions in each object's body frame

Vertex positions in world frame

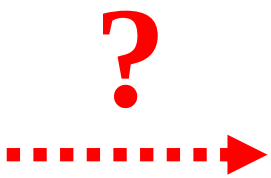
Vertex positions in 2D viewport

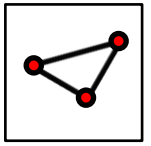
1. Placing objects



p_1
p_2
p_3

Mp_1
Mp_2
Mp_3





We need to introduce the concept of a "camera" looking at the "scene".

Then the next steps would be:

2. Placing a "camera"
3. Selecting its "lens"
4. Displaying on a "cinema screen"

In Terms of CG Transformation,

- 1. Placing objects

→ **Modeling transformation**

- 2. Placing a "camera"

→ **Viewing transformation**

- 3. Selecting its "lens"

→ **Projection transformation**

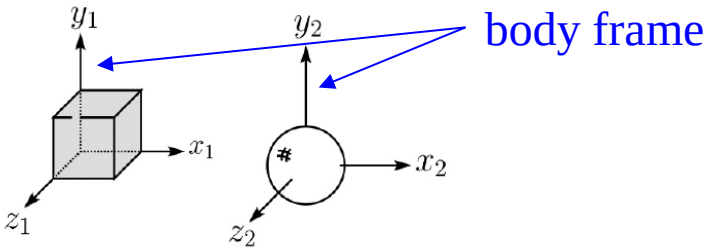
- 4. Displaying on a "cinema screen"

→ **Viewport transformation**

- All these transformations just work by **matrix multiplications**!

Vertex Processing (Transformation Pipeline)

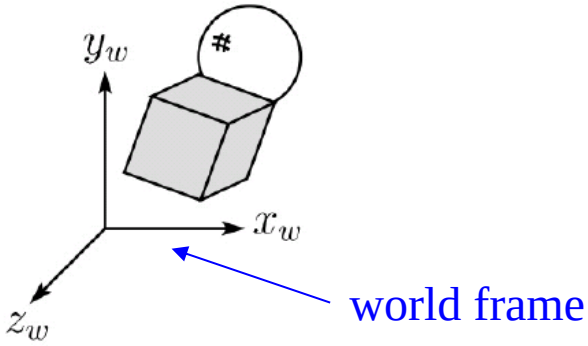
Object space



body frame

Translate, scale, rotate, ... any affine transformations

(What we've covered in prev. lectures)



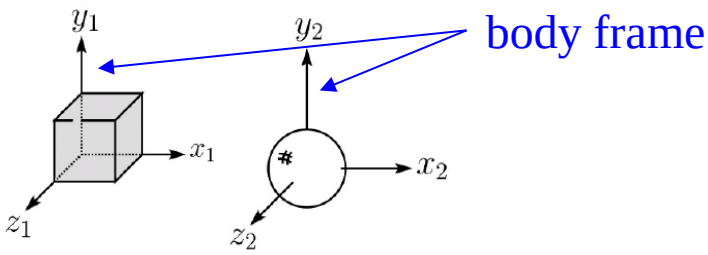
world frame

World space

* The images are from the slides of Prof. Jinxiang Chai (Texas A&M University): http://faculty.cs.tamu.edu/jchai/csce441_2016spring/lectures.html

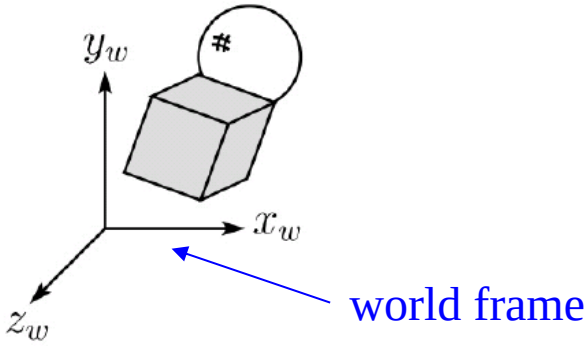
Vertex Processing (Transformation Pipeline)

Object space



body frame

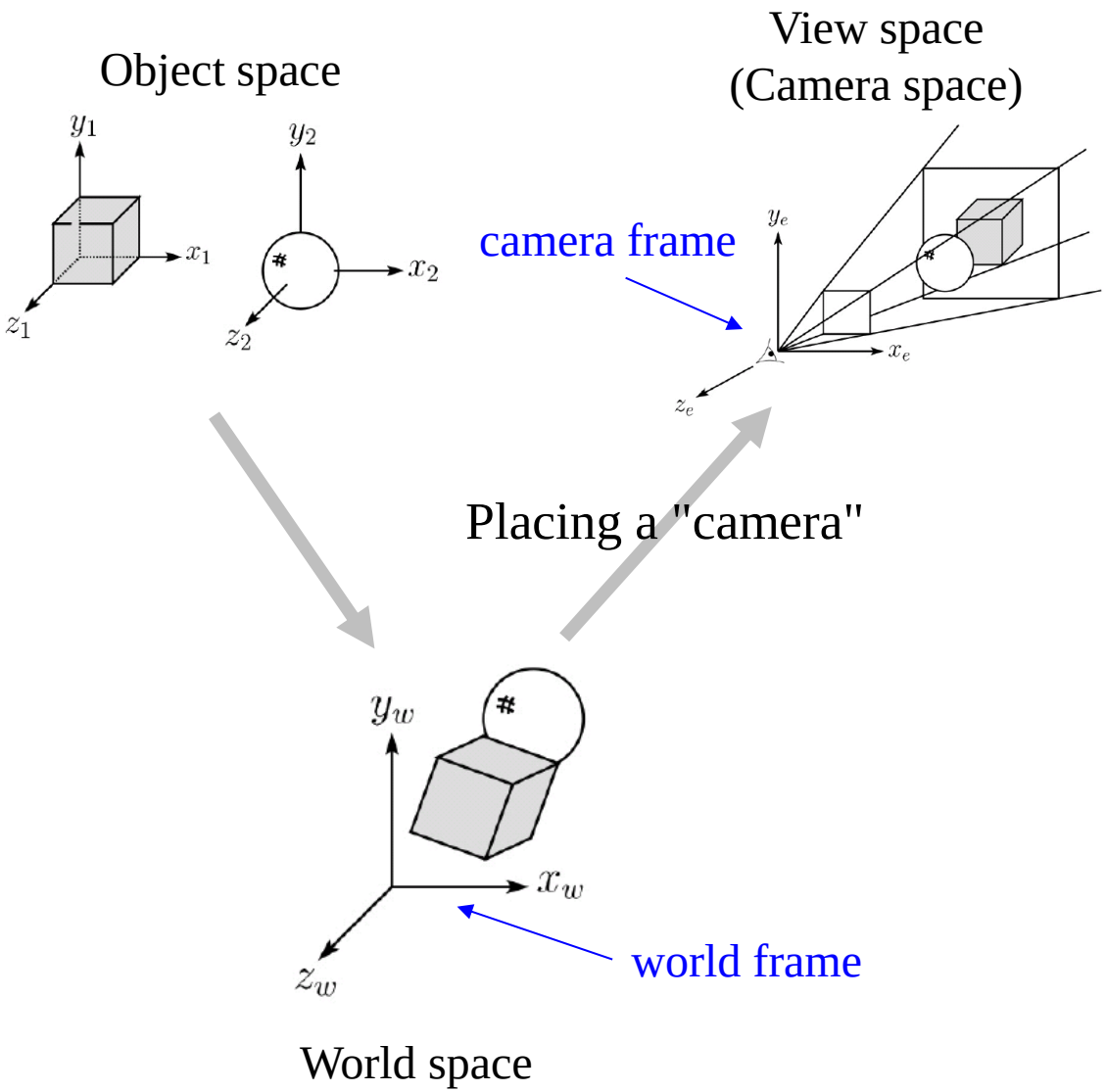
Modeling transformation



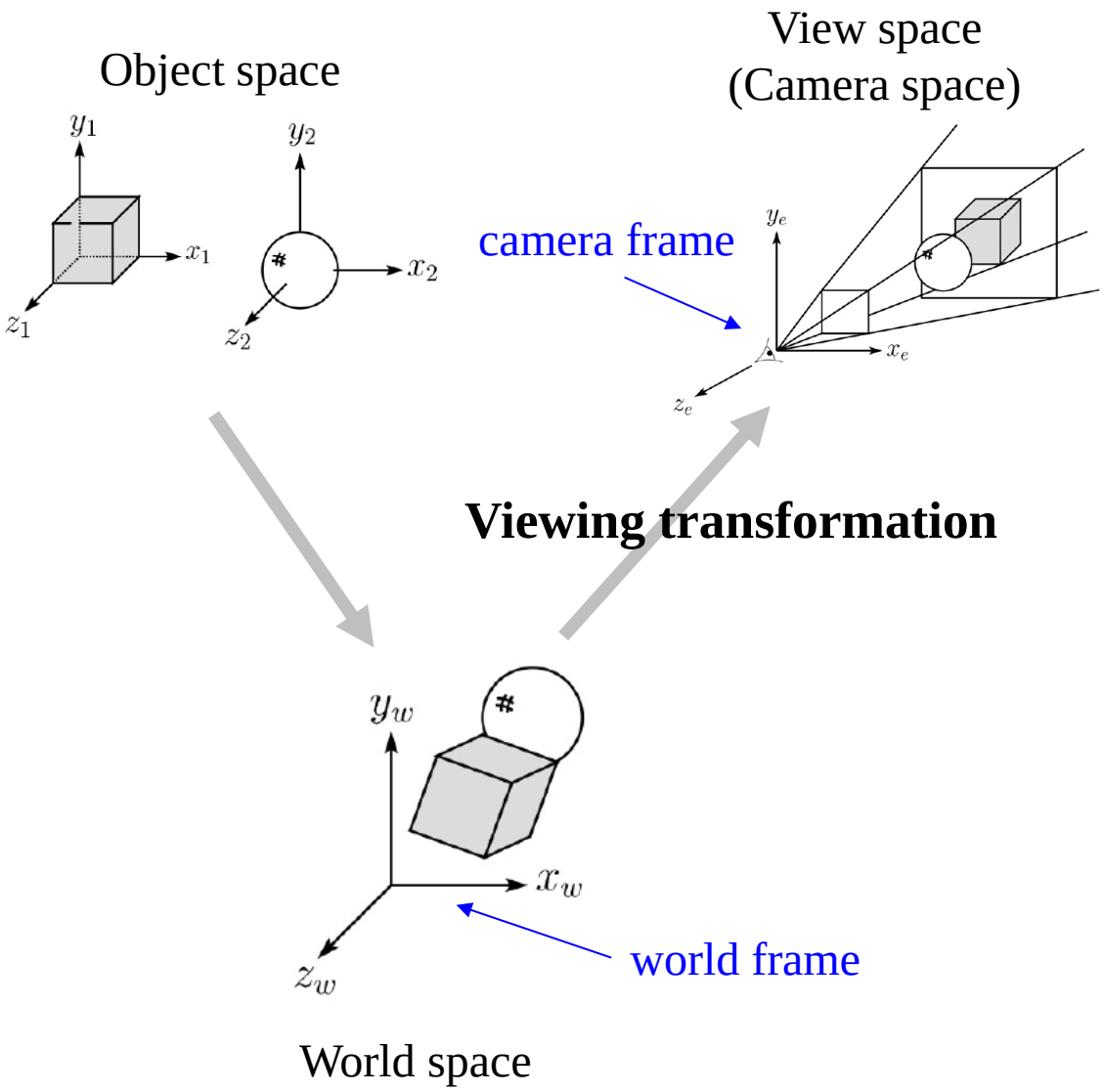
world frame

World space

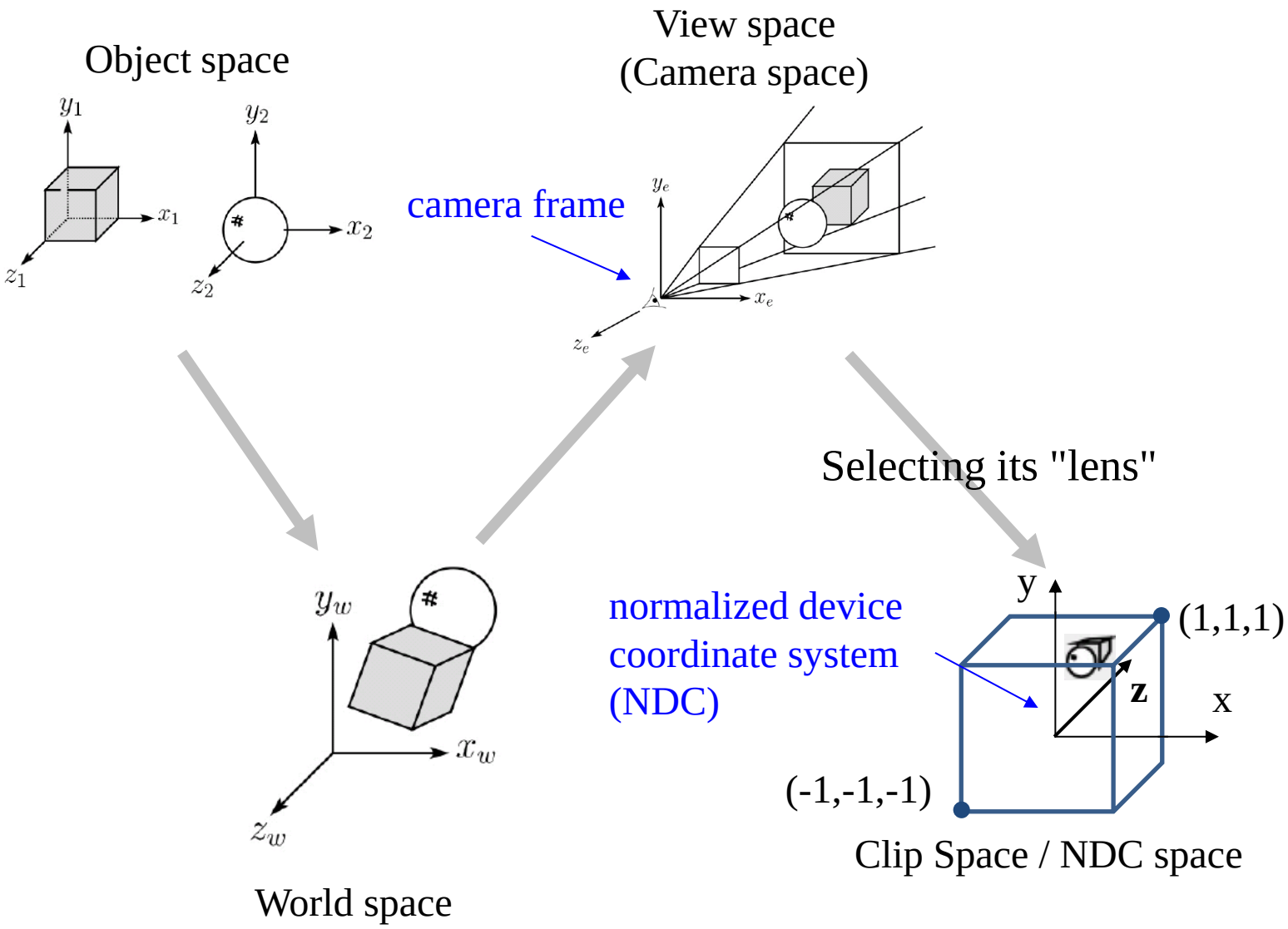
Vertex Processing (Transformation Pipeline)



Vertex Processing (Transformation Pipeline)

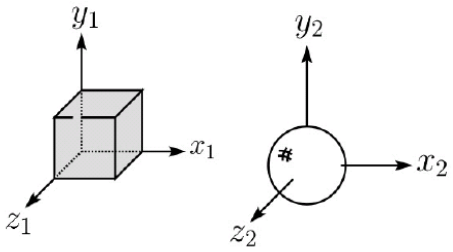


Vertex Processing (Transformation Pipeline)

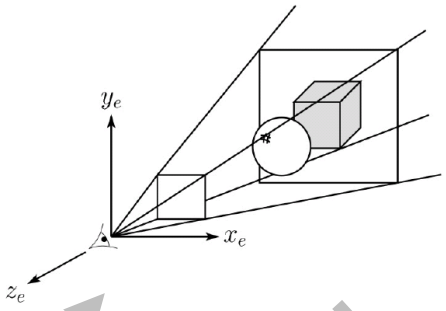


Vertex Processing (Transformation Pipeline)

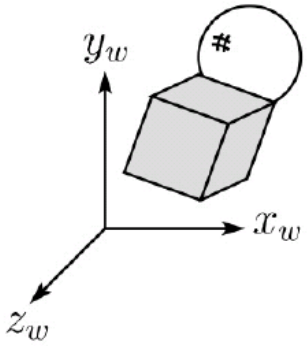
Object space



View space
(Camera space)

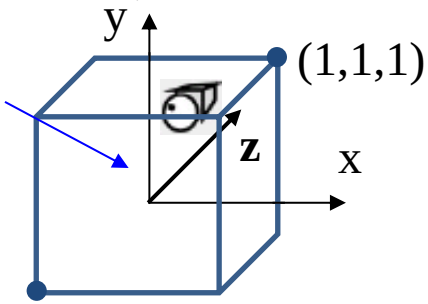


Projection transformation



World space

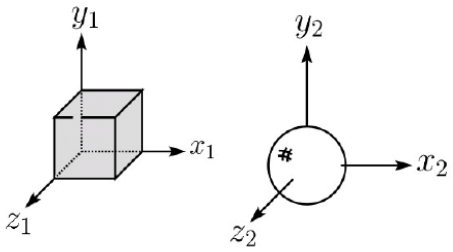
normalized device
coordinate system
(NDC)



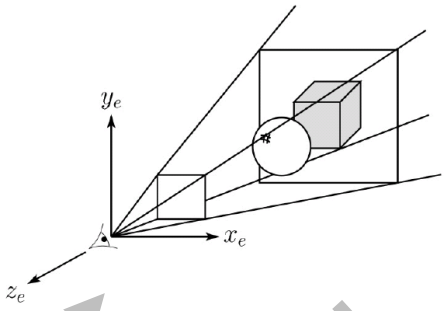
Clip Space / NDC space

Vertex Processing (Transformation Pipeline)

Object space

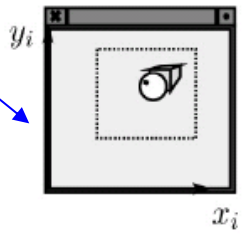


View space
(Camera space)

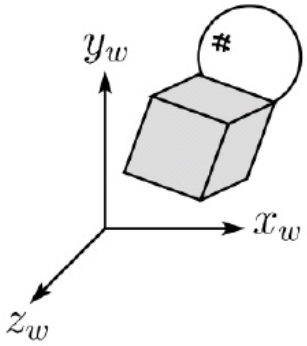


screen
coordinate
system

Screen space
(Image space)

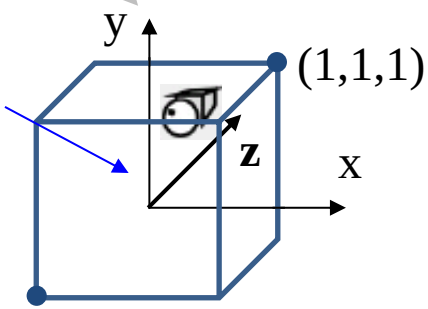


Displaying on a
"cinema screen"



World space

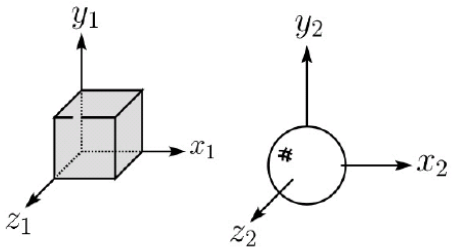
normalized device
coordinate system
(NDC)



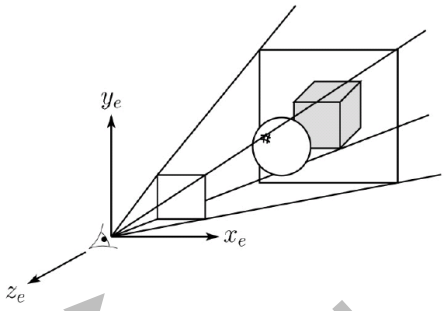
Clip Space / NDC space

Vertex Processing (Transformation Pipeline)

Object space

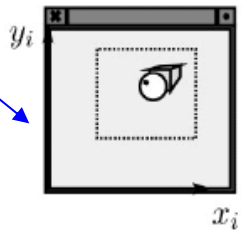


View space
(Camera space)

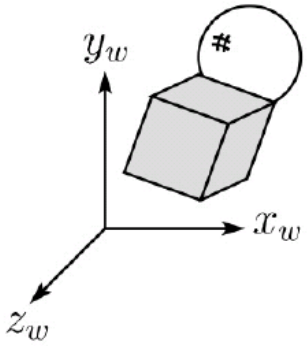


screen
coordinate
system

Screen space
(Image space)

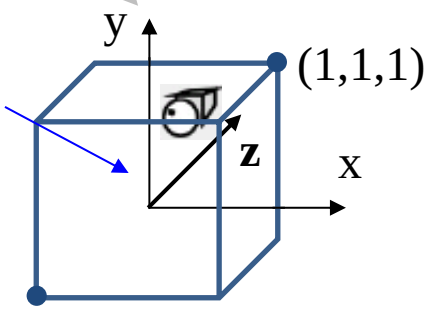


Viewport transformation



World space

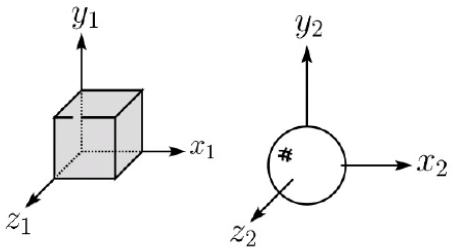
normalized device
coordinate system
(NDC)



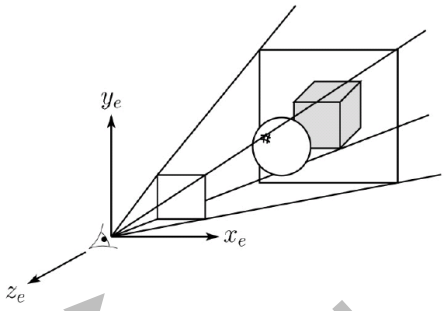
Clip Space / NDC space

Vertex Processing (Transformation Pipeline)

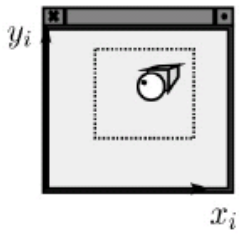
Object space



View space
(Camera space)



Screen space
(Image space)

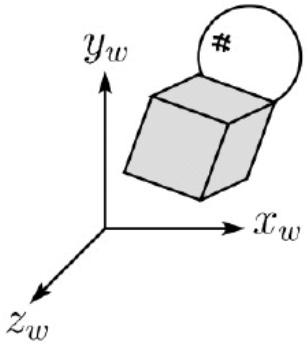


Modeling transformation

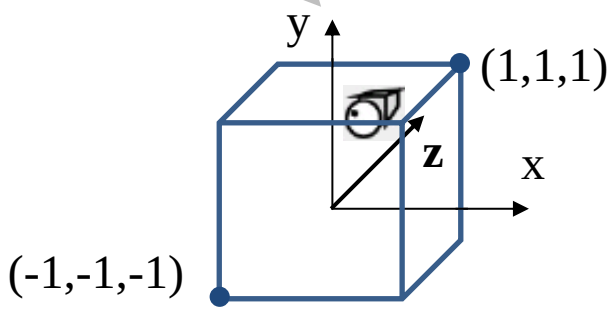
Viewing transformation

Projection transformation

Viewport transformation



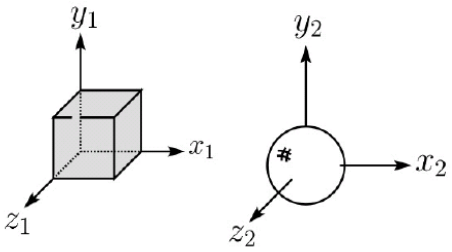
World space



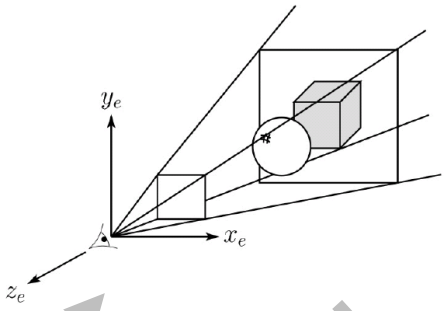
Clip Space / NDC space

Vertex Processing (Transformation Pipeline)

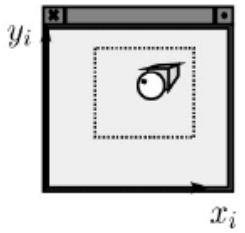
Object space



View space
(Camera space)



Screen space
(Image space)

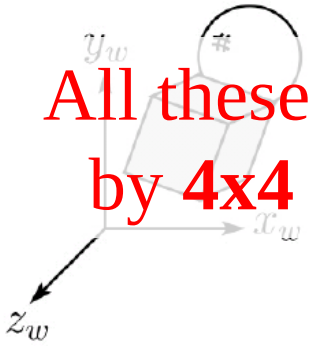


Modeling transformation

Viewing transformation

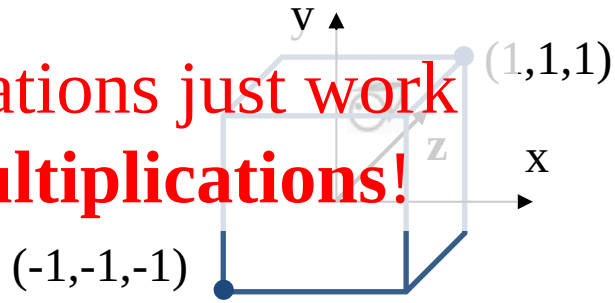
Projection transformation

Viewport transformation



World space

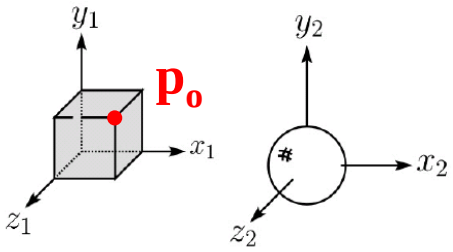
All these transformations just work by **4x4 matrix multiplications!**



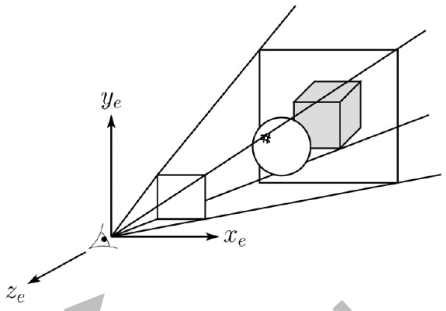
Clip Space / NDC space

Vertex Processing (Transformation Pipeline)

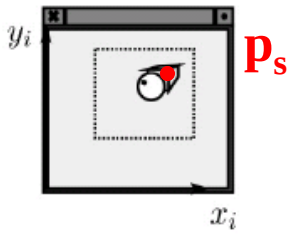
Object space



View space
(Camera space)



Screen space
(Image space)



Modeling transformation : M

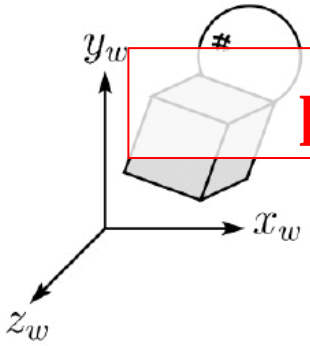
Viewing transformation : V

Projection transformation : P

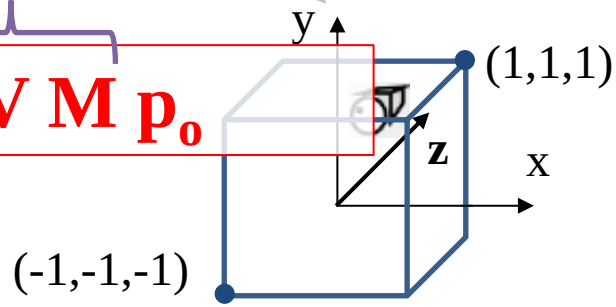
Viewport transformation : T_{vp}

MVP matrix

$$p_s = T_{vp}\, P\, V\, M\, p_o$$



World space

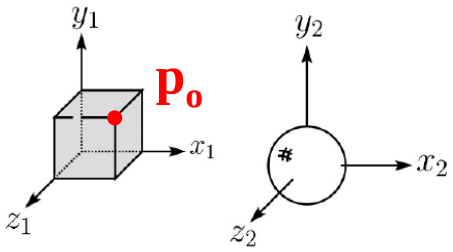


Clip Space / NDC space

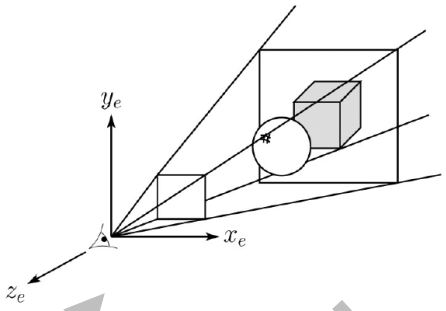
Modeling Transformation

Modeling Transformation

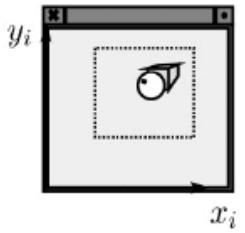
Object space



View space
(Camera space)



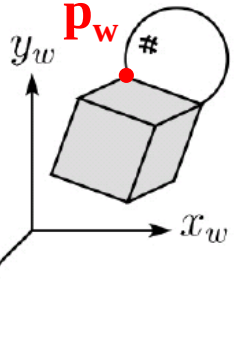
Screen space
(Image space)



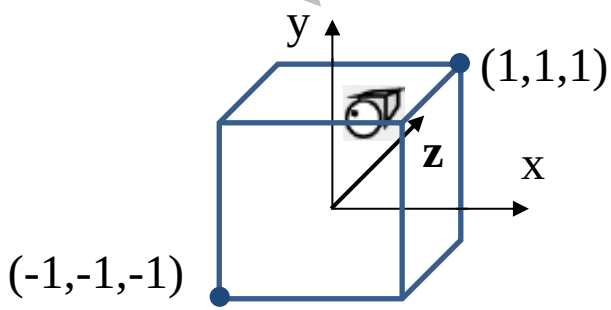
Modeling transformation : M

$p_w = M\ p_o$

Note that this arrow direction means...

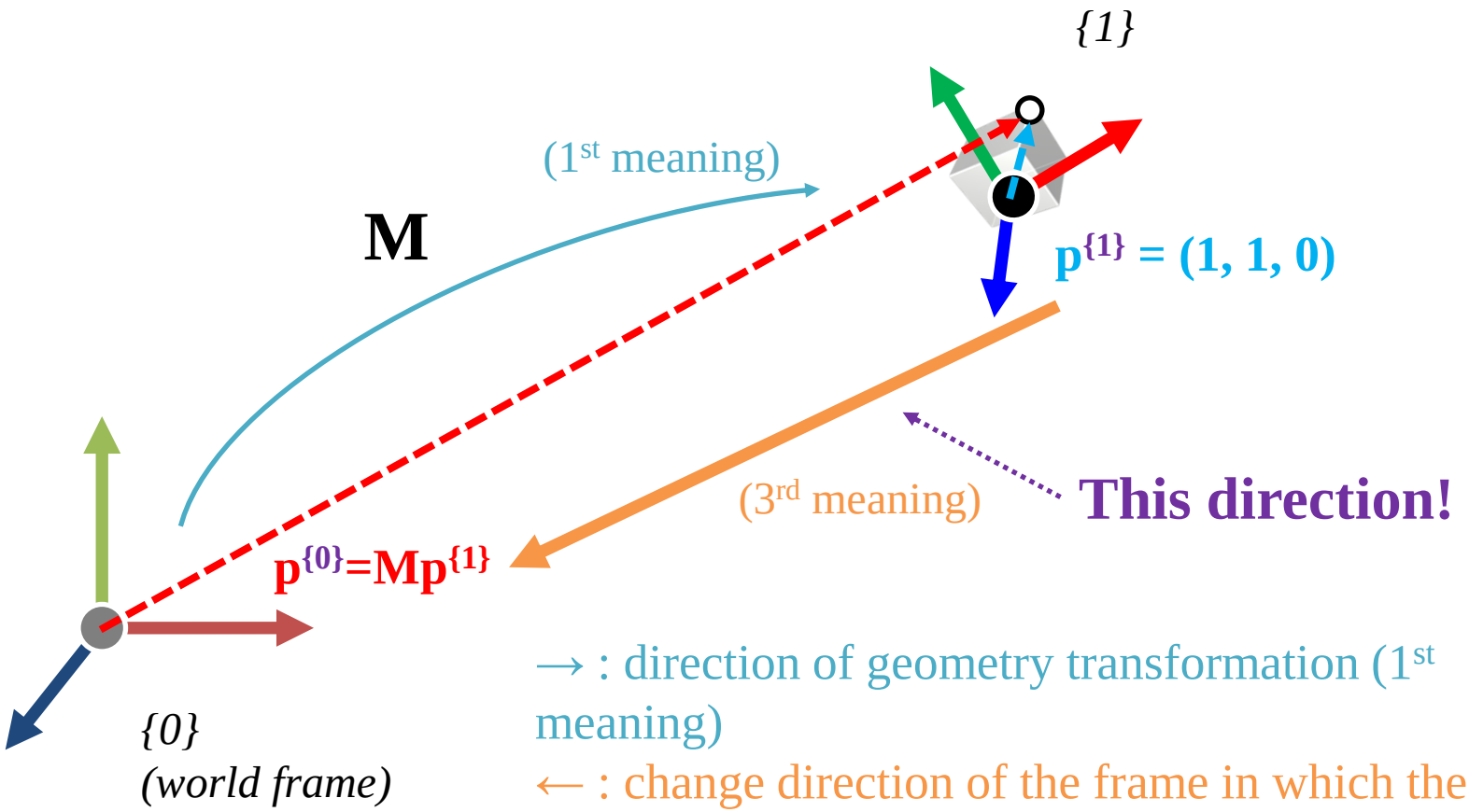


World space



Clip Space / NDC space

Recall: Directions of the "arrow"

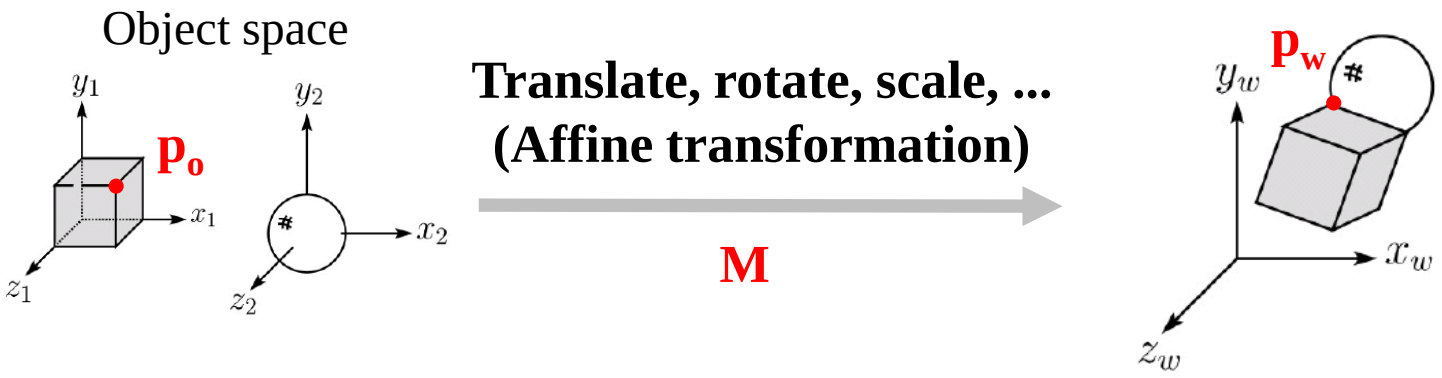


→ : direction of geometry transformation (1st meaning)

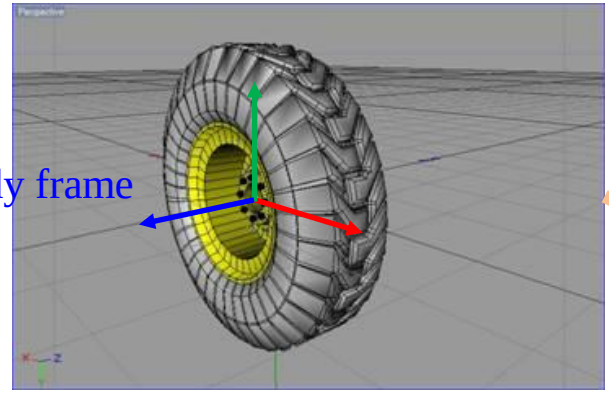
← : change direction of the frame in which the point is described (3rd meaning) - {1} → {0}

Modeling Transformation

- An object is defined in the **object's body frame**.
- The modeling transformation (from object space to world space) is represented by the *modeling matrix*, **M.**
- This **M** is a composite affine transformations that we've covered so far.



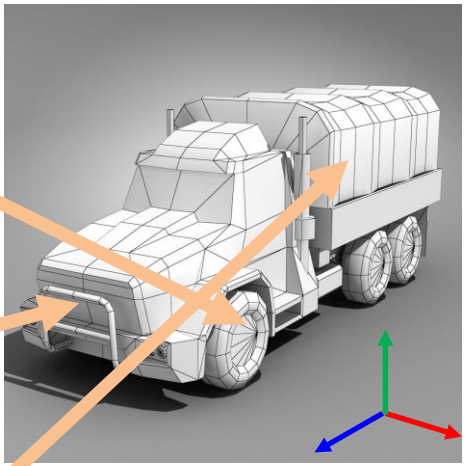
Wheel object space



body frame

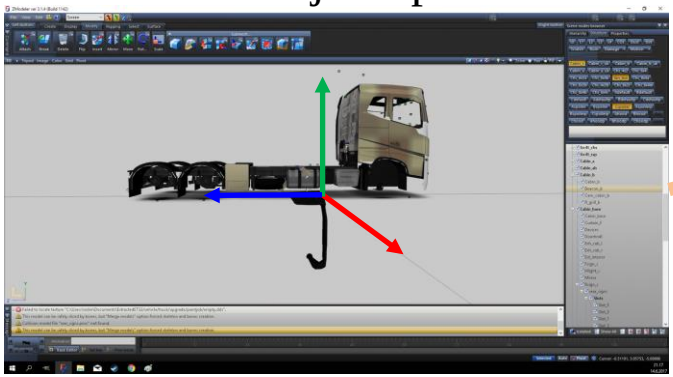
M^{wheel}

World space



world frame

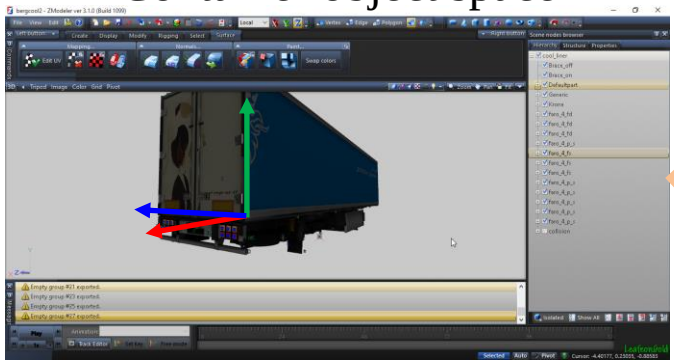
Cab object space



M^{cab}

$M^{container}$

Container object space



Quiz 1

- Go to https://www.slido.com/
- Join #**cg-ys**
- Click "Polls"

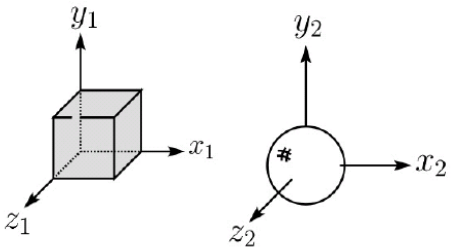
- Submit your answer in the following format:
  - **Student ID: Your answer**
  - **e.g. 2021123456: 4.0**

- Note that your quiz answer must be submitted **in the above format** to be counted for attendance.

Viewing Transformation

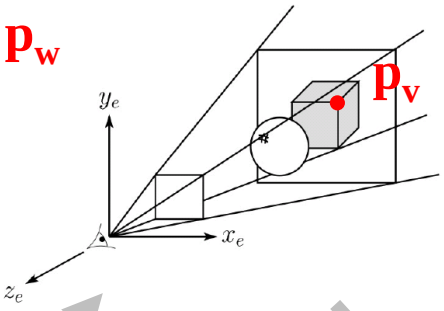
Viewing Transformation

Object space

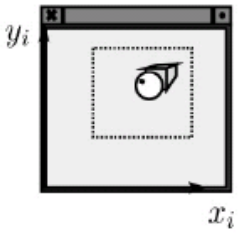


View space
(Camera space)

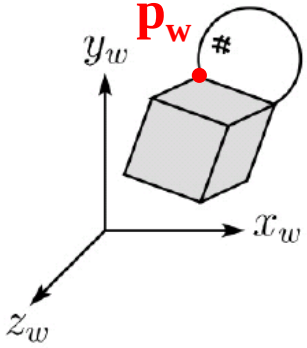
$\mathbf{p_v = V\,p_w}$



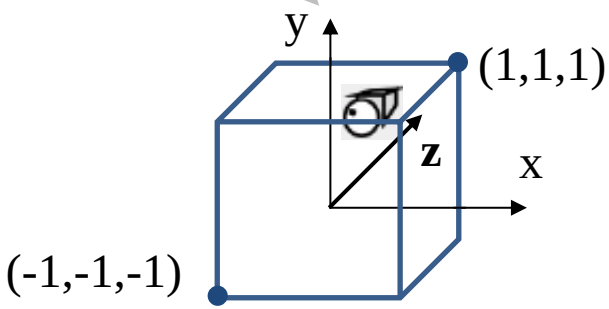
Screen space
(Image space)



Viewing transformation : V



World space



Clip Space / NDC space

Recall that...

- 1. Placing objects

→ **Modeling transformation**

- 2. Placing a "camera"

→ **Viewing transformation**

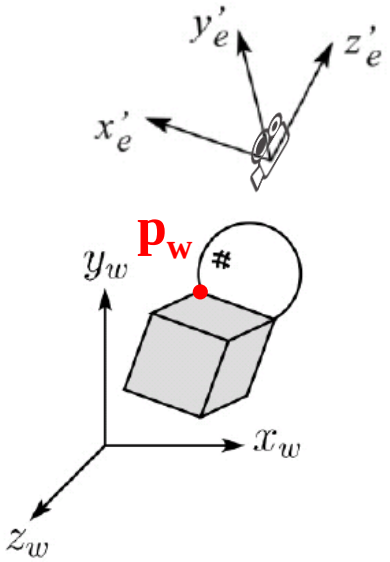
- 3. Selecting its "lens"

→ **Projection transformation**

- 4. Displaying on a "cinema screen"

→ **Viewport transformation**

Viewing Transformation

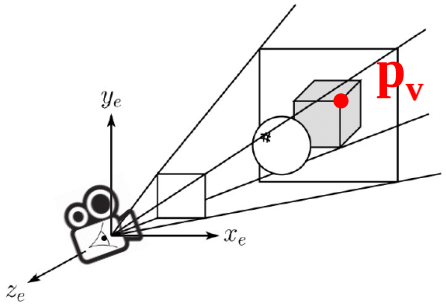


World space

Translate & rotate
(Rigid transformation)

V

View space
(Camera space)



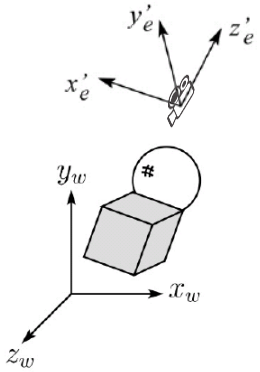
- The viewing transformation (from world space to view space) is represented by the *viewing matrix*, **V.**

- This is a rigid transformation.

Viewing Transformation

- Goal: Expressing all object vertices in the camera's coordinate system (*camera frame*).

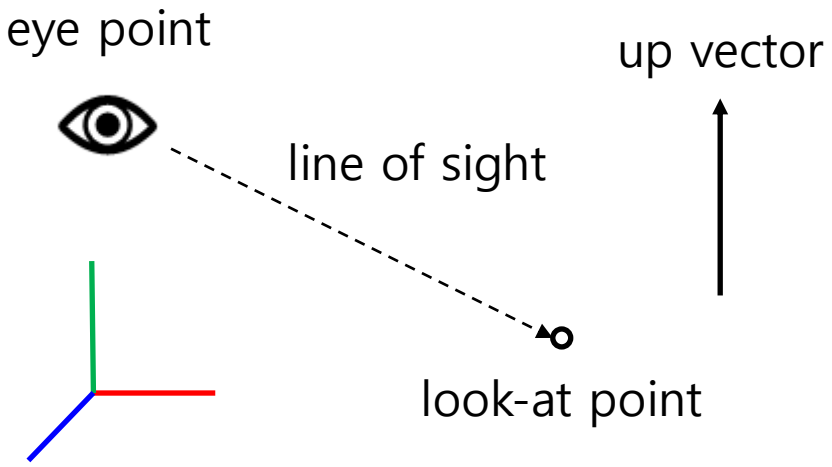
- To do that, we need to define the *camera frame* (w.r.t. world frame).



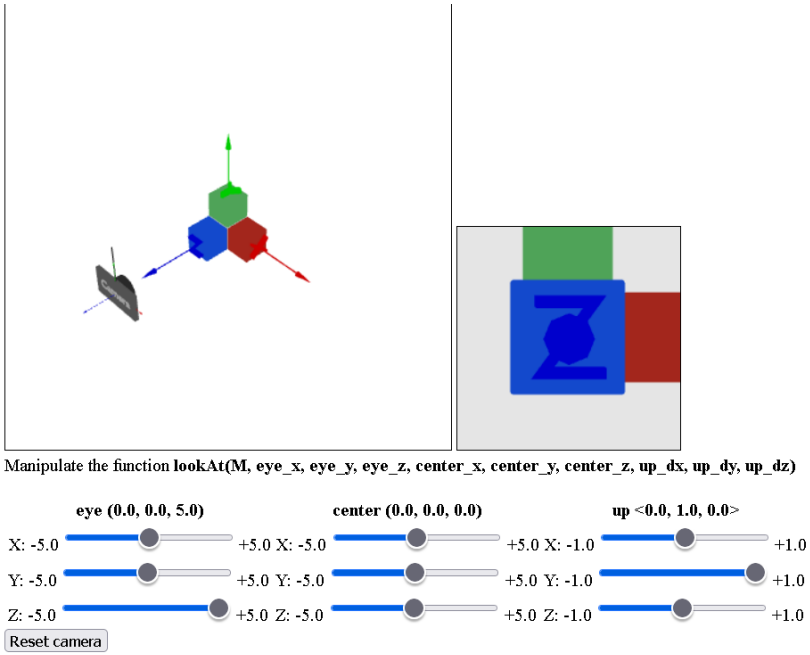
- Defining the *camera frame* is the same as determining *the position and orientation of the camera.*

Defining Camera Frame 1 - "LookAt"

- Many ways to specify the camera's position & orientation.
- One intuitive method is "*lookat" function* that uses:
- **Eye point**
  - Camera position
- **Look-at point**
  - Camera target position
- **Up vector**
  - Roughly defines which direction is *up*



[Demo] LookAt Function



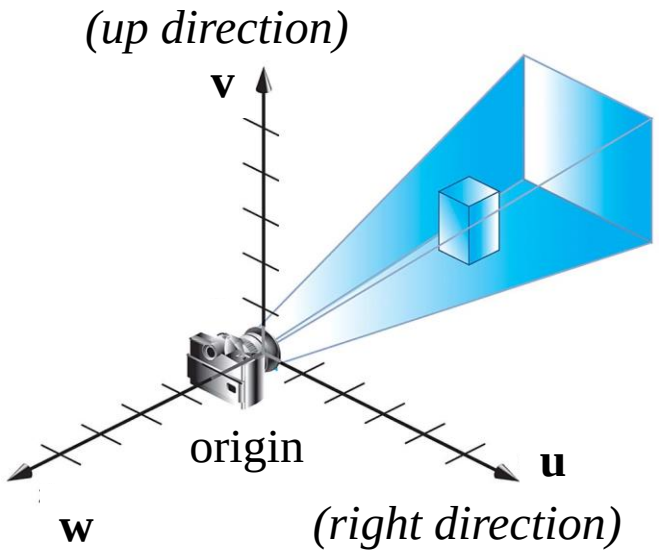
http://learnwebgl.brown37.net/07_cameras/camera_lookat/camera_lookat.html

- Observe the 3D scene (left) and rendered view (right) while changing eye, center, up by dragging sliders.

Defining Camera Frame 1 - "LookAt"

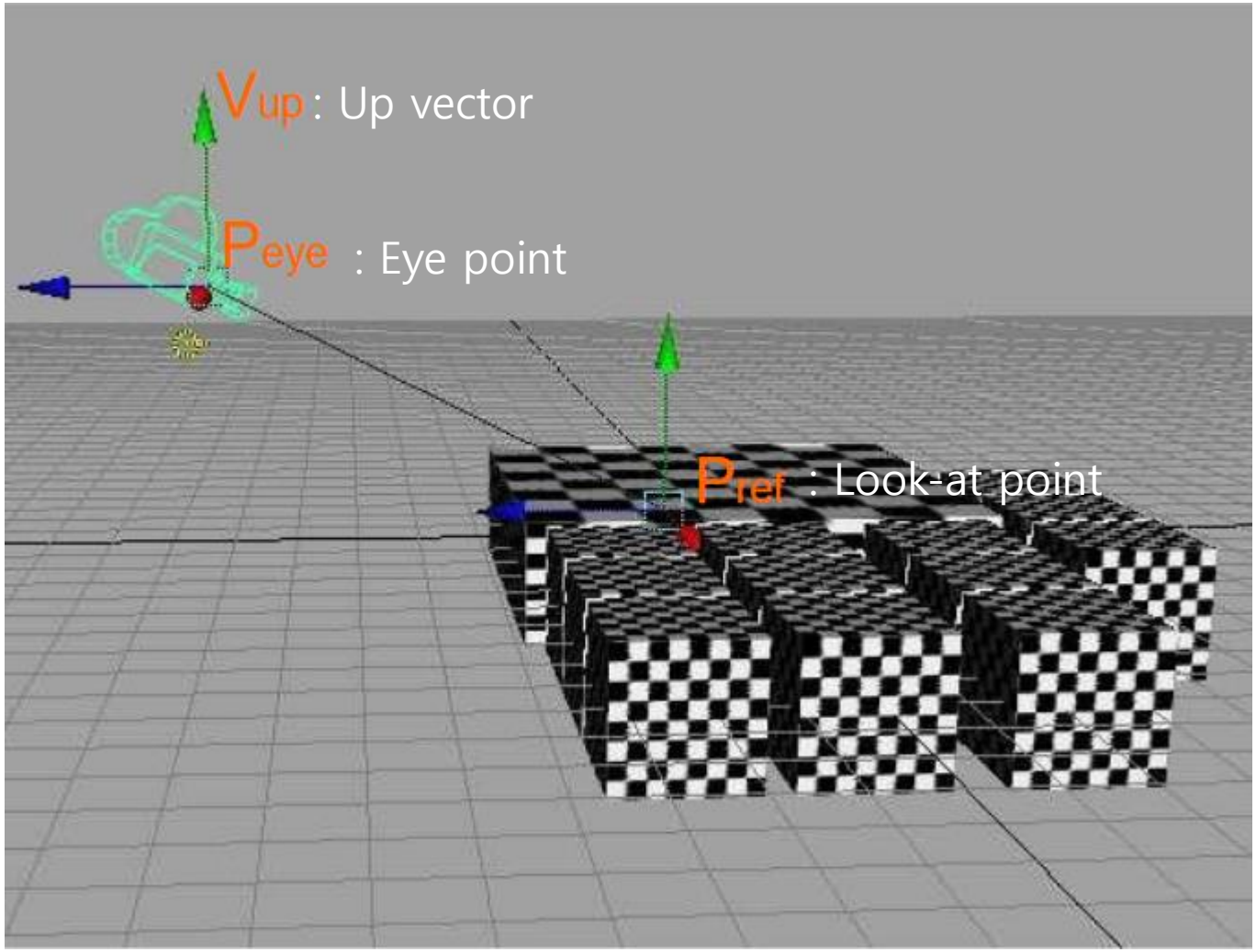
- From the given **eye point**, **look-at point**, **up vector**, we can compute the **camera frame**.

- Note that **u, v, w** are commonly used for camera coordinates axes instead of x, y, z.



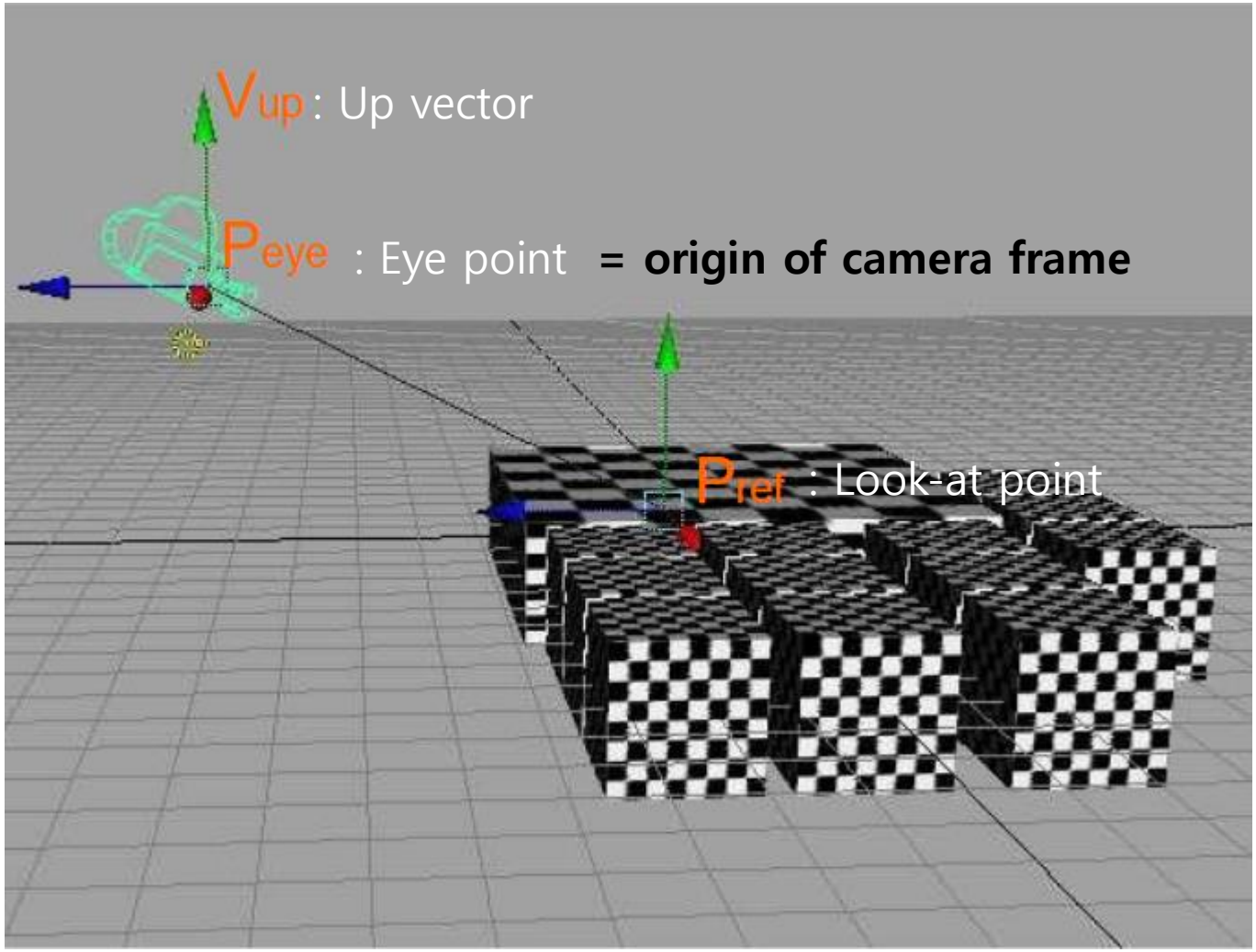
- To define the camera frame, we need to find:
- **u, v, w** vectors
- **origin** point

Given Eye point, Look-at point, Up vector,

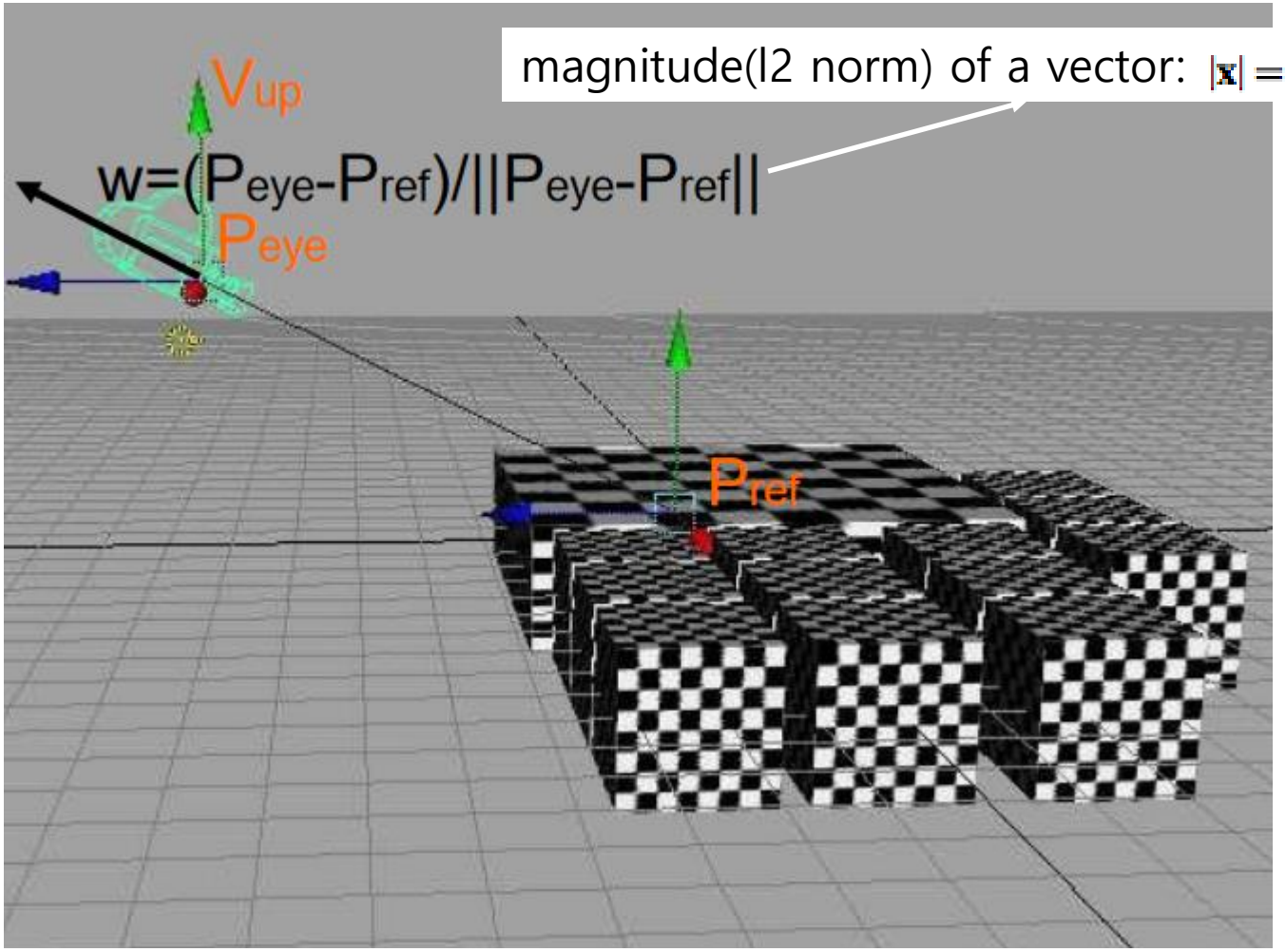


* The base images are from the slides of Prof. Karan Singh's (University of Toronto): http://www.dgp.toronto.edu/~karan/courses/418/

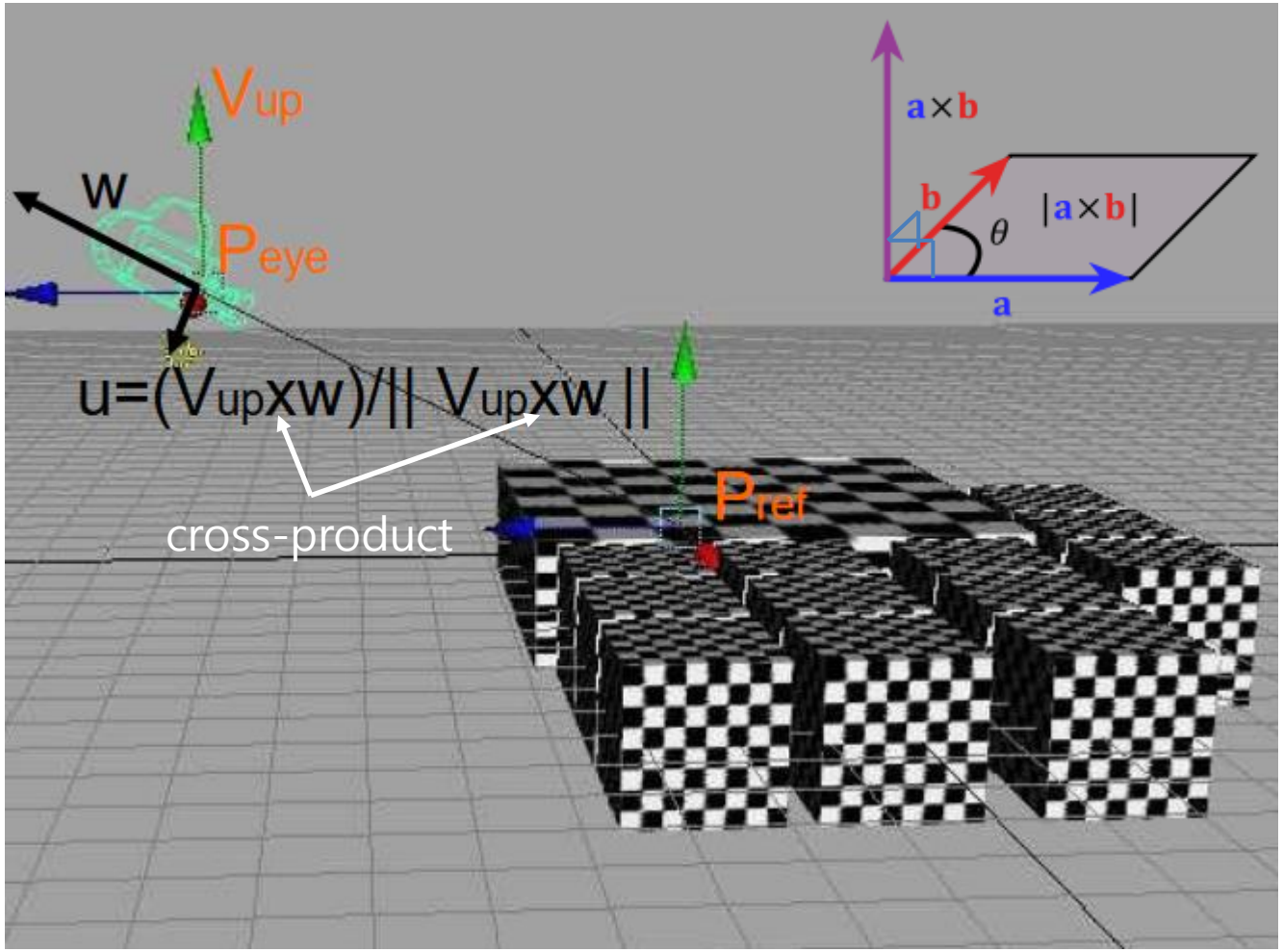
Getting origin point



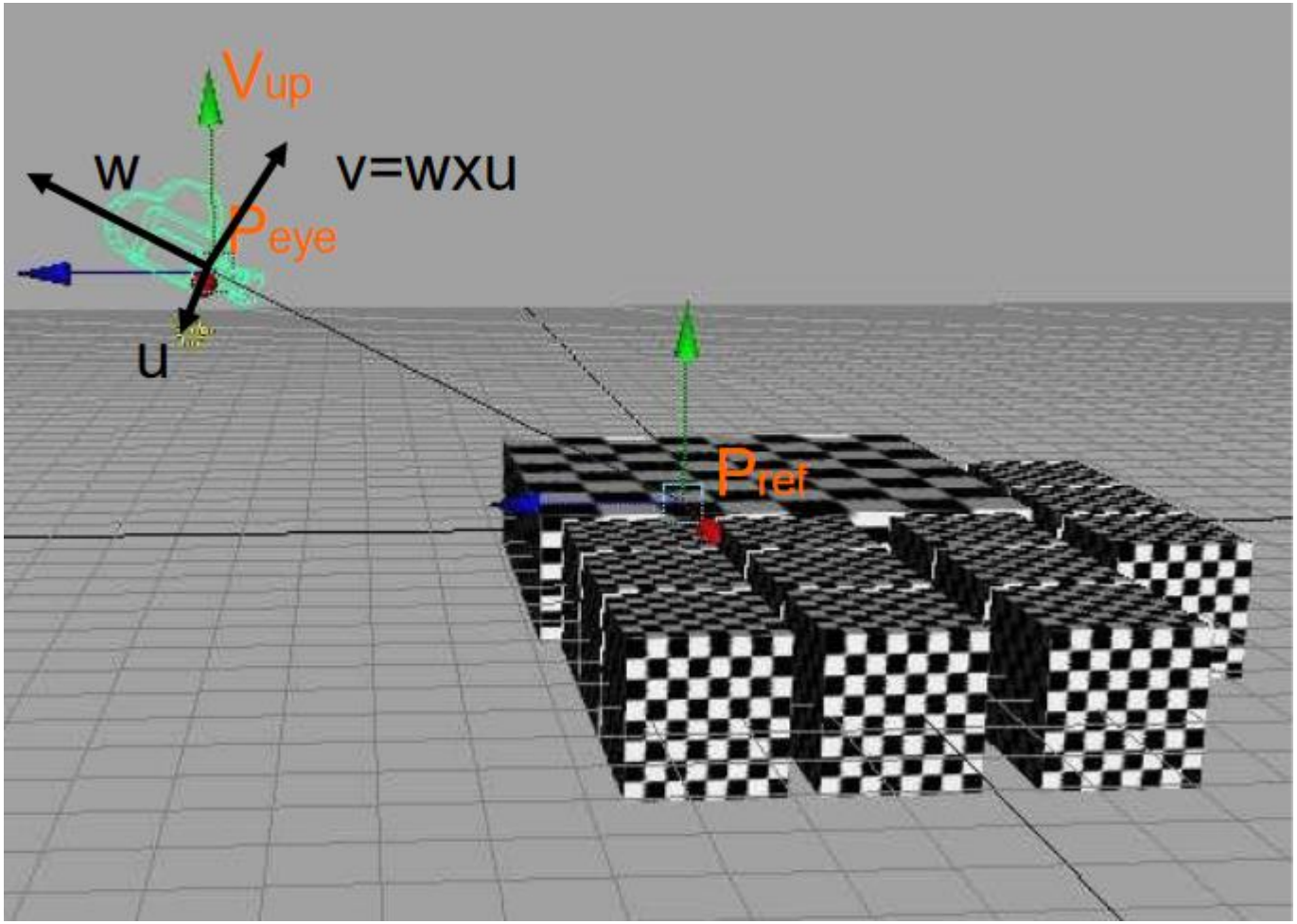
Getting “w” axis vector



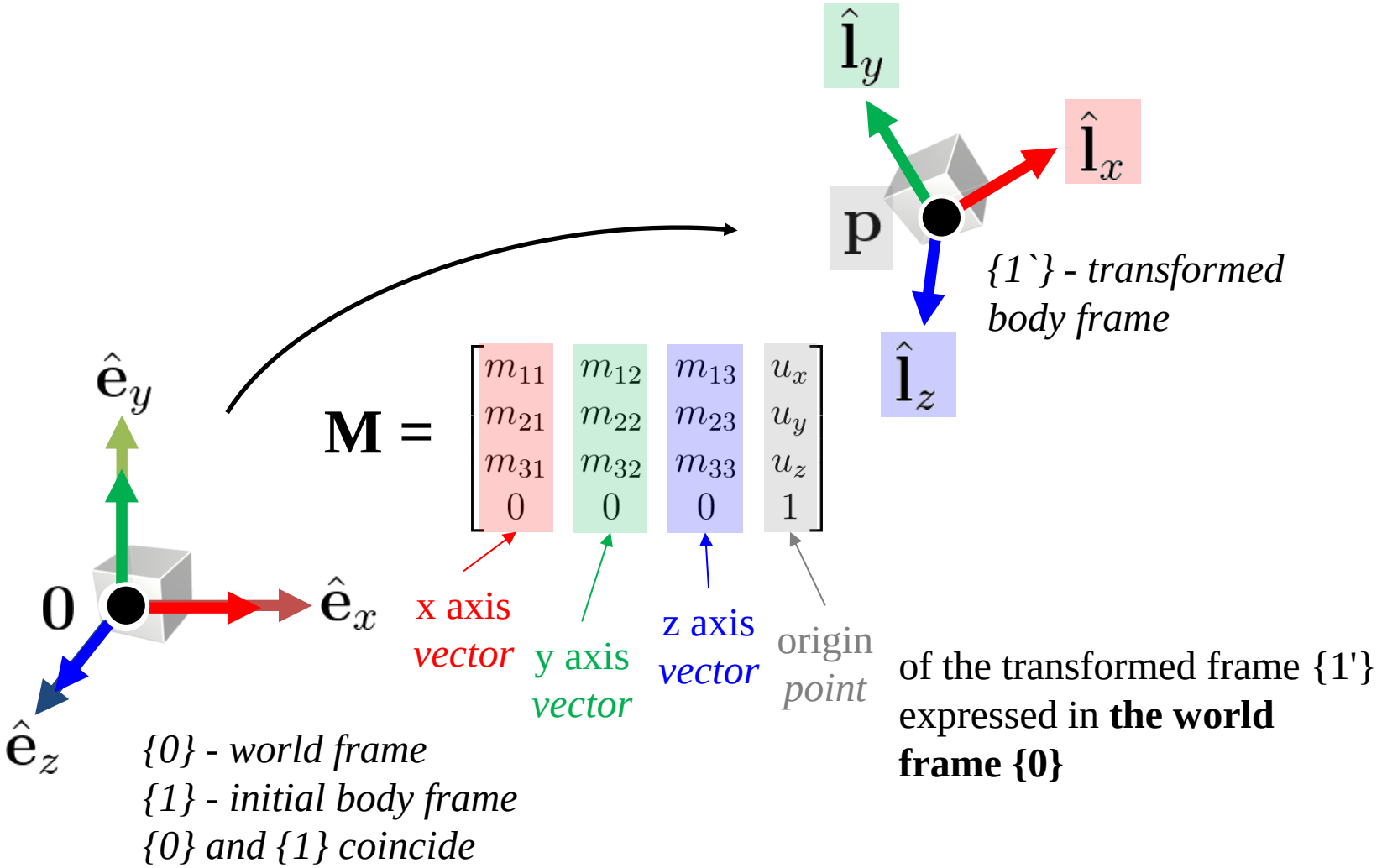
Getting “u” axis vector



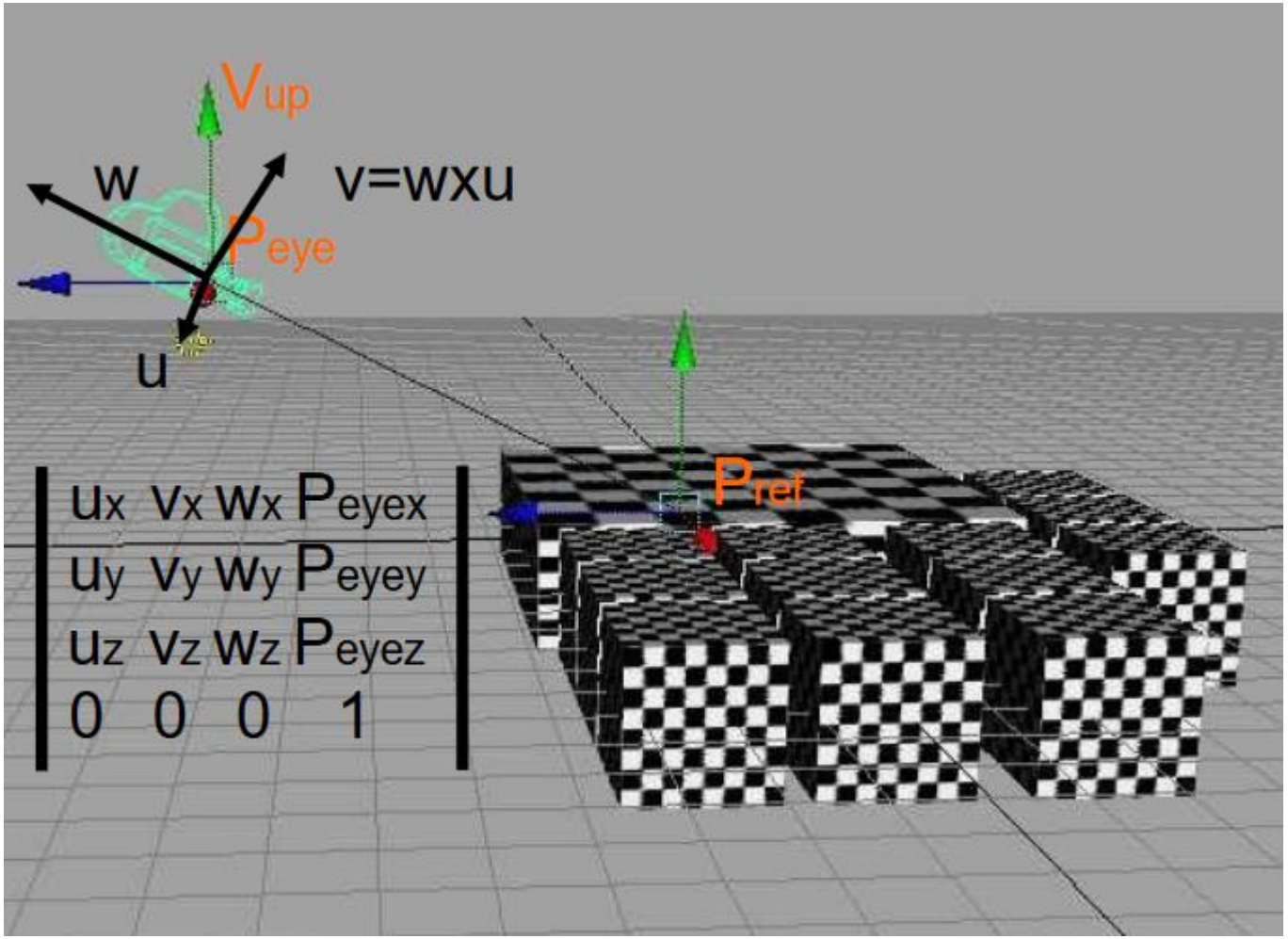
Getting “v” axis vector



Recall: 2) Affine Transformation Matrix defines an Affine Frame w.r.t. World Frame

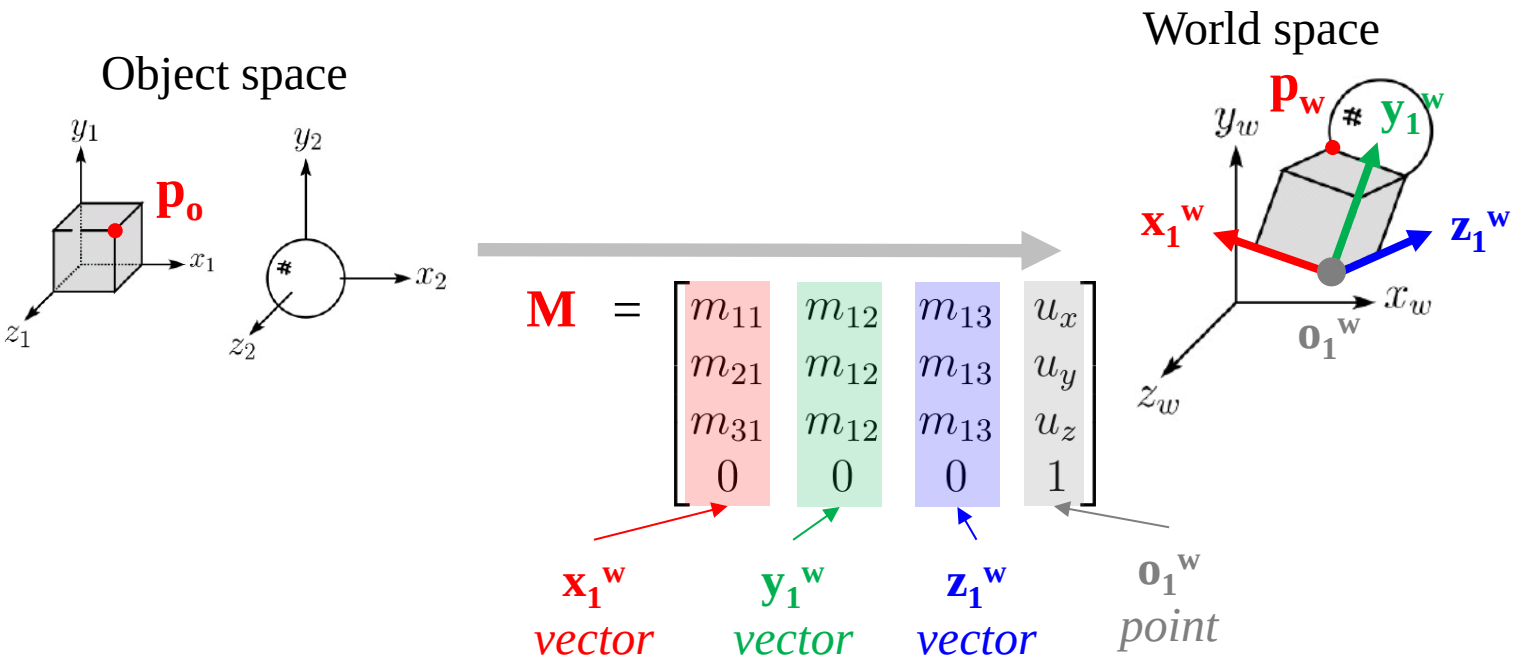


Thus, the Camera Frame is defined by



How can we get viewing matrix V from the camera frame?

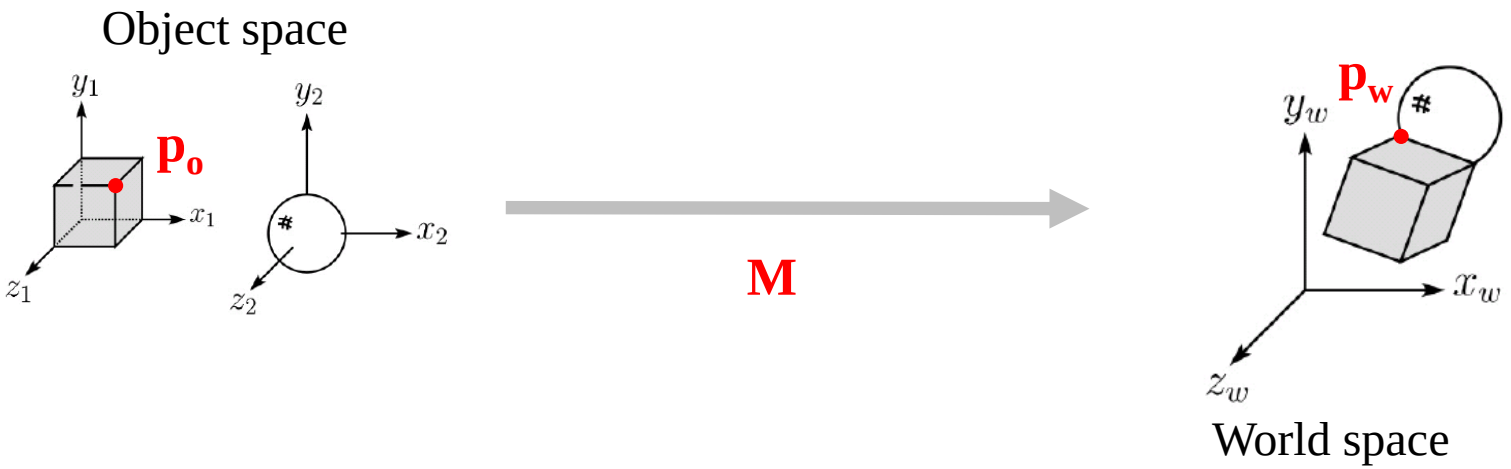
- Recall the modeling transformation:



: The axis vectors and origin point of the **object's body frame represented in the world frame**

How can we get viewing matrix V from the camera frame?

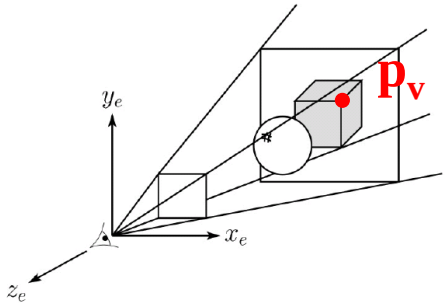
- If we replace *object space* to *camera space*, what should be the transformation matrix?



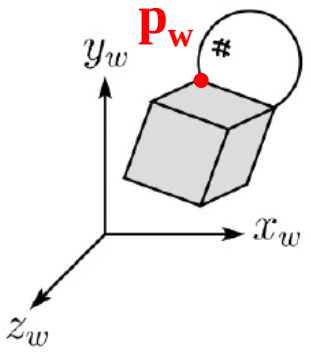
How can we get viewing matrix V from the camera frame?

- If we replace *object space* to *camera space*, what should be the transformation matrix?

View space
(Camera space)



?

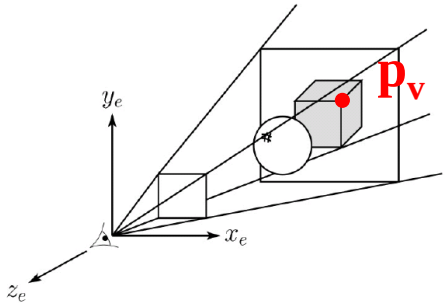


World space

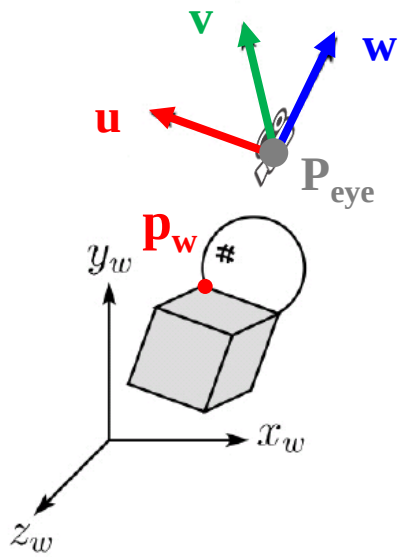
How can we get viewing matrix V from the camera frame?

- If we replace *object space* to *camera space*, what should be the transformation matrix?

View space
(Camera space)



$$\begin{bmatrix} u_x & v_x & w_x & P_{eyex} \\ u_y & v_y & w_y & P_{eyey} \\ u_z & v_z & w_z & P_{eyez} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

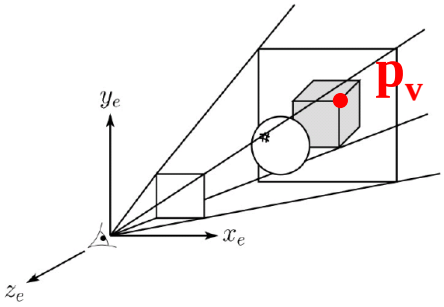


World space

: The axis vectors and origin point of the **camera frame represented in the world frame**

Viewing Transformation is the Opposite Direction

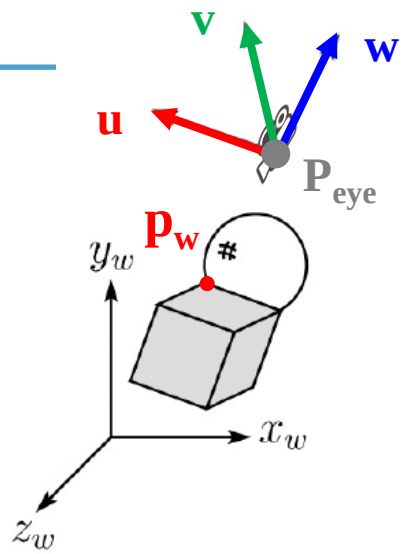
View space
(Camera space)



V

$$\begin{vmatrix} u_x & v_x & w_x & P_{eyex} \\ u_y & v_y & w_y & P_{eyey} \\ u_z & v_z & w_z & P_{eyez} \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Rigid transformation matrix



World space

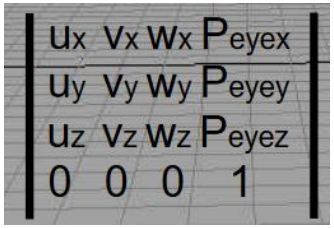
$$V = \begin{vmatrix} u_x & v_x & w_x & P_{eyex} \\ u_y & v_y & w_y & P_{eyey} \\ u_z & v_z & w_z & P_{eyez} \\ 0 & 0 & 0 & 1 \end{vmatrix}^{-1}$$

Inverting Rigid Transformation Matrix

(R: 3×3 rotation matrix, t: 3×1 translation vector)

$$T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \Rightarrow T^{-1} = \begin{bmatrix} R^T & -R^T t \\ 0 & 1 \end{bmatrix}$$

- For our camera frame matrix,



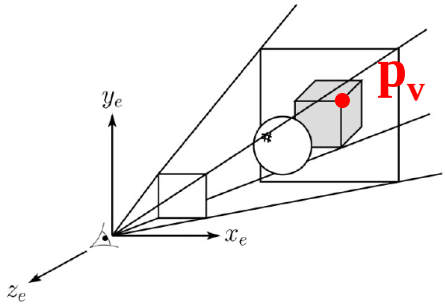
$$R = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} \quad \mathbf{t} = \begin{bmatrix} P_{\text{eyex}} \\ P_{\text{eyey}} \\ P_{\text{eyez}} \end{bmatrix}$$

$$\Rightarrow \quad R^T = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}$$

$$-R^T \cdot \mathbf{t} = - \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix} \begin{bmatrix} P_{\text{eyex}} \\ P_{\text{eyey}} \\ P_{\text{eyez}} \end{bmatrix} = \begin{bmatrix} -(\mathbf{u} \cdot \mathbf{t}) \\ -(\mathbf{v} \cdot \mathbf{t}) \\ -(\mathbf{w} \cdot \mathbf{t}) \end{bmatrix}$$

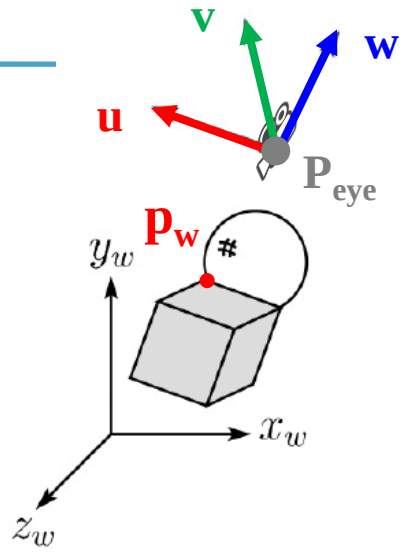
Viewing Transformation is the Opposite Direction

View space
(Camera space)



V

$$\begin{vmatrix} u_x & v_x & w_x & P_{eyex} \\ u_y & v_y & w_y & P_{eyey} \\ u_z & v_z & w_z & P_{eyez} \\ 0 & 0 & 0 & 1 \end{vmatrix}$$



World space

$$\mathbf{V} = \begin{vmatrix} u_x & v_x & w_x & P_{eyex} \\ u_y & v_y & w_y & P_{eyey} \\ u_z & v_z & w_z & P_{eyez} \\ 0 & 0 & 0 & 1 \end{vmatrix}^{-1} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{u}\cdot\mathbf{p}_{eye} \\ v_x & v_y & v_z & -\mathbf{v}\cdot\mathbf{p}_{eye} \\ w_x & w_y & w_z & -\mathbf{w}\cdot\mathbf{p}_{eye} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

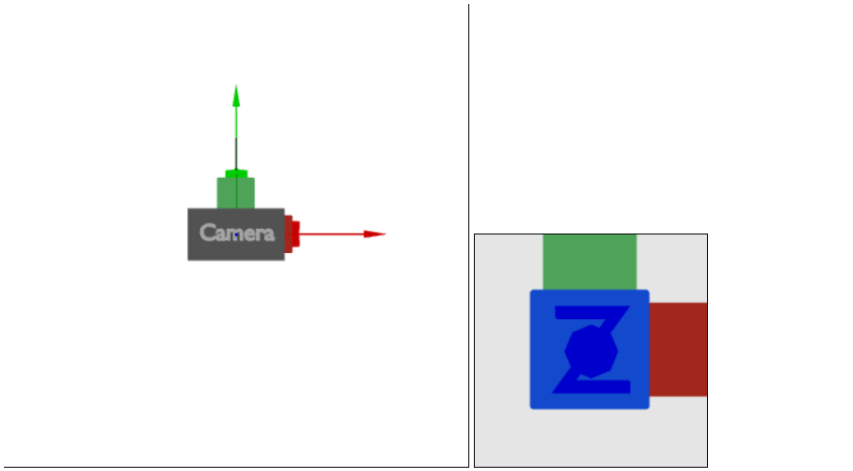
Defining Camera Frame 2 - Translate & Rotate

- "LookAt" is not the only way to specify the camera's position and orientation.

- You can just "translate" and "rotate" the camera instead.

- The camera frame is then just defined by these rigid transformation matrices.

[Demo] Translate & Rotate Camera

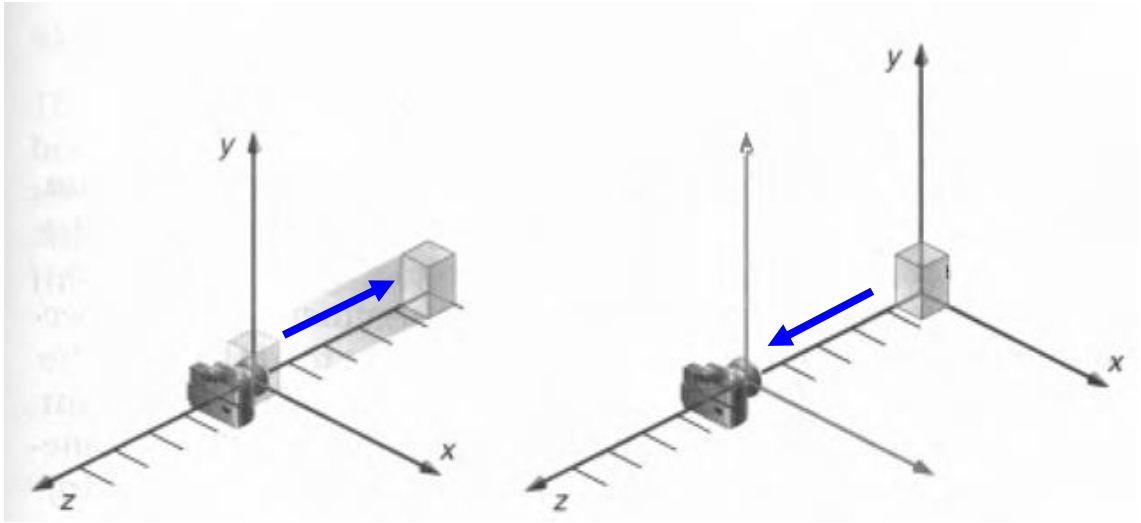


http://learnwebgl.brown37.net/07_cameras/camera_trunk_axes/camera_trunk_axes.html

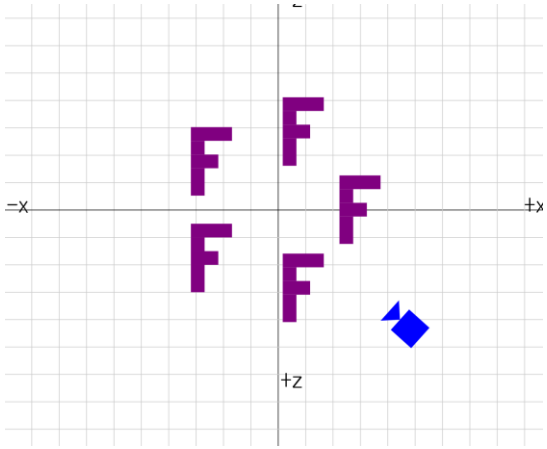
- Observe the 3D scene (left) and rendered view (right) while translating the camera with "eye" sliders and rotating it with axis selection and "angle" slider.

Moving Camera vs. Moving World

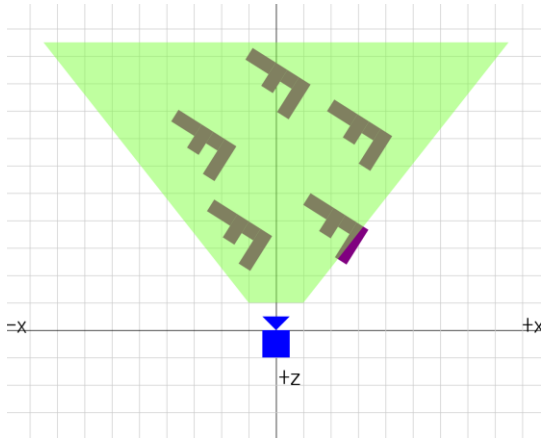
- Actually, these are two **equivalent operations.**
- Translate camera by (1, 0, 2) == Translate world by (-1, 0, -2)
- Rotate camera by 60° about y == Rotate world by -60° about y



[Demo] Moving Camera vs. Moving World



https://webglfundamentals.org/webgl/lessons/resources/camera-move-camera.html?mode=0



https://webglfundamentals.org/webgl/lessons/resources/camera-move-camera.html?mode=2

- (Left) Moving camera
- (Center) Moving world

Lab Session

- Now, let's start the lab today.